Generalized Transfer Factors for Granular Beds

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SEVERAL of the heat and mass transfer and the pressure drop characteristics of packed and fluidized beds, the subject of many studies, are compared, and a generalized correlation of the data is suggested. Areas in which further work is desirable are also mentioned.

DISCUSSION OF PREVIOUS WORK

Following the earlier theoretical work of Reynolds, Prandtl, and Taylor, Chilton and Colburn (*3*) showed that heat and mass transfer data for turbulent flow in pipes could be related to the friction factor by the empirical equations:

$$j_H = \frac{h}{C_p G}\left(\frac{C_p\mu}{k}\right)^{2/3} = \frac{1}{2}f$$

$$j_D = \frac{k_g P_b M_m}{G}\left(\frac{\mu}{\rho D_v}\right)^{2/3} = \frac{1}{2}f$$

These j factors have been most useful correlating factors. Modern mass transfer texts (*16, 18*) and the literature describe the development of these factors and give examples of their use for mass, heat, and momentum transfer correlations for empty pipes, for wetted wall towers, and to some extent for single spheres. However, the lack of agreement shown in the working equations proposed by various authors indicates the necessity of an analysis of the problem based on more fundamental considerations of flow effect in various cases before a valid generalized correlation may be derived.

Chu (*4, 16*) and his students have provided an excellent summary of the use of the j factor for correlating mass transfer data for fixed and fluidized beds. Data for both types of beds correlated as a smooth curve when the j factor was plotted against a superficial Reynolds number corrected for voidage as $DG/\mu(1-\epsilon)$. Their correlation was therefore similar to that proposed by Gamson (*5*), which involved a correlation of $jD/(1-\epsilon)^{0.2}$ against a modified Reynolds number.

Studies on the transfer of heat between particles and the fluid in packed and fluidized beds have been less numerous because of the experimental difficulties in obtaining particle surface temperatures. However, the results of several experiments have been included.

Previous analyses of pressure drop in fixed beds have considered either that the bed is a system of interconnected channels or that individual particles have contributed to the over-all pressure drop. Fluidized beds have been treated by both mechanisms as expanded fixed beds. In including pressure drop data for fixed and fluidized beds and for transported particles in the correlation proposed here, the model summing the contributions of individual particles has been used. The transition from fixed bed to fluidized bed to transported bed may be considered a single continuous operation of granular beds, in which the fixed bed is referred to as the basic state. This can be easily observed by increasing the velocity of a fluid flowing through a granular bed, starting from the fixed bed state.

After examining correlations in the literature, notably those of Chu and Gamson, the following questions arose regarding heat, mass, and momentum transfer studies for particles in fluids.

In the Chu and Gamson correlations, is the correction for voidage, $1 - \epsilon$, the most suitable? Since this correction approaches zero as a bed expands, a correlation using this type of correction would not be expected to hold for fluidized beds at high voidages and for transported particles. It was questioned also whether the slip velocity of particles in a fluid should not enter a correlation.

Can the j factors be modified so that correlations indicate that mass transfer coefficients and pressure drops are increasing with increased flow through the bed? The Froessling equation, the Sieder-Tate equation, and similar correlations for heat and mass transfer show increasing transfer coefficients as Reynolds number increases. The j factors, on the other hand, decrease as the Reynolds number is increased because they include the mass transfer rate in their denominator.

Could such a correlation be made more inclusive? The purpose of this study was to attempt to set up a correlation which could be made to include single particles and transported beds, where the voidage of bed approaches unity.

Modified transfer factors have been obtained by multiplying the conventional j factors by the Reynolds number. These transfer factors appear to demonstrate a continuity between fixed and fluidized beds and single particles and perhaps transported particles. For fluidized beds, the use of a "hypothetical relative Reynolds number" as the j factor multiplier introduced the correction factor

$$\left(\frac{u + u_t}{u}\right)\left(\frac{\epsilon - \epsilon_0}{\epsilon_0}\right)\epsilon_0$$

It is believed that this correction factor is an improvement over the $1 - \epsilon$ factor previously used in the Reynolds number.

The proposed correlation uses the superficial particle Reynolds number as a common correlating variable for all cases. The graphs appear to reflect the dependencies of the transfer coefficients on the fluid velocity.

The approximate values of film coefficients for both mass and heat transfer and friction coefficients in granular beds can be estimated by using the proposed Equations 12 and 13 from conventional Reynolds number.

HEAT AND MASS TRANSFER IN FIXED OR PACKED BEDS

The modified correlating group for fixed beds becomes a standard for other situations. The modified correlating groups are obtained by multiplying the conventional j factors by the actual Reynolds number giving

$$T_H = j_h \times N_{\mathrm{Re}} = \frac{h}{C_p G}\left(\frac{C_p\mu}{k}\right)^{2/3}\frac{D_p G}{\mu} = \frac{hD_p}{k} \Big/ \left(\frac{C_p\mu}{k}\right)^{1/3} = \mathrm{Nu}/\mathrm{Pr}^{1/3} \quad (1)$$

$$T_D = j_D \times N_{\mathrm{Re}} = \frac{k_g P_b M_m}{G}\left(\frac{\mu}{\rho D_v}\right)^{2/3}\frac{D_p G}{\mu} = \frac{k_g P_b M_m D_p}{D_v \rho} \Big/ \left(\frac{\mu}{\rho D_v}\right)^{1/3} = \mathrm{Nu}'/\mathrm{Sc}^{1/3} \quad (2)$$

where G is the actual velocity of the fluid which is equal to the superficial velocity divided by the void fraction of bed. These are correlated against the superficial Reynolds number. The factor called the T factor here, is a function of transfer coefficient and physical properties only. This group is already used to plot data, but is not as popular as j factors.

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HEAT AND MASS TRANSFER IN FLUIDIZED BEDS

The fluid flow in a fluidized bed requires a slip velocity correction. This has been incorporated into the Reynolds number used to multiply the j factor as indicated.

An exact expression cannnot be written for the motion of particle in a fluid where the relative velocity between the particle and the fluid is not small.

A slip velocity, V_s, may be defined as the difference between the average velocity of the particle and that of the fluid in the z direction as: $V_s\ (\overline{u}_{solid} - \overline{u}_{fluid})_z$. In a vertical granular bed, the flow of the solid particles and of the fluid is countercurrent and both are in the direction of the pipe axis. Because of gravity and strong longitudinal turbulence in comparison to the negligible radical turbulence, the motion of each particle in the bed is primarily in a vertical direction. Where the particles are in free vertical motion without collision of one with another, the average slip velocity, $\overline{V}_{s\infty}$, will be: $\overline{V}_{s\infty} = u_t - (-u) = u_t + u$. The sign of $\overline{V}_{s\infty}$ determines the direction, upward or downward, of the motion of individual particles in a bed or in an eddy.

At time instant, t, the vertical velocity of a particle in a homogeneous fluid is proportional to the distance it has traveled. Considering a fluidized bed of unit volume or unit height, the maximum distance particles are free to move about is equal to $\epsilon - \epsilon_0$ length unit, and the actual average distance the particles may travel before and after consecutive collisions is equal to $(\epsilon - \epsilon_0)\ \epsilon$ length unit. Since the average slip velocity of particles in free vertical motion is: $\overline{V}_{s\infty} = u_t + u$, the actual average slip velocity of particles in a bed in which the same average number of particles repeat upward and downward motion at the same velocity may be derived using the fixed bed state as the reference state of zero slip velocity as:

$$\overline{V}_{sr} = \frac{(u + u_t)(\epsilon - \epsilon_0)}{\epsilon_0}\ \epsilon$$

or (3)

$$N_{\text{Re}}' = \frac{D_p \rho u}{\mu} \left(\frac{u + u_t}{u}\right) \left(\frac{\epsilon - \epsilon_0}{\epsilon_0}\right) \epsilon$$

In the correction $\left(\frac{u + u_t}{u}\right) \left(\frac{\epsilon - \epsilon_0}{\epsilon_0}\right) \epsilon$, in which the terms $\left(\frac{u + u_t}{u}\right) \left(\frac{\epsilon - \epsilon_0}{\epsilon_0}\right)$ have been called a "stage of fluidization constant," u_t is the terminal, free-falling velocity of the particles in the still fluid, u is the superficial velocity of fluid, and ϵ_0 is the void fraction of a bed just at the onset of fluidization.

If u_t is not available, it may be estimated by plotting ϵ against u and extrapolating to the u value corresponding to $\epsilon = 1.0$. $\left(\frac{u + u_t}{u}\right) \left(\frac{\epsilon - \epsilon_0}{\epsilon_0}\right)$ apparently reaches a constant value of approximately 3 ± 0.15, when particulate fluidization has been reached (Figure 1); this usually occurs at void fraction above 0.65.

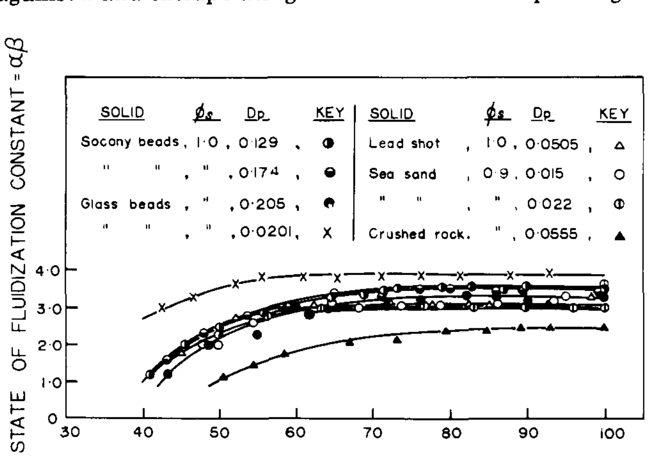


Figure 1. Stage of fluidization constant vs. ϵ

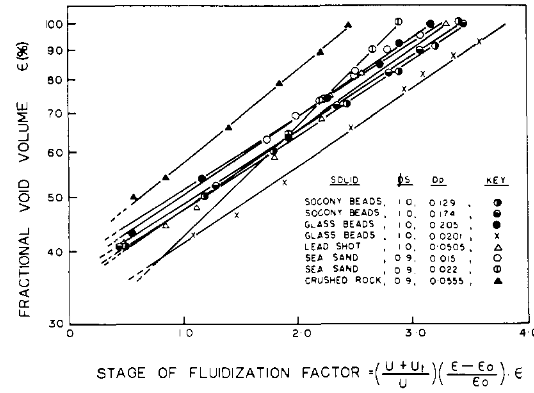


Figure 2. Stage of fluidization factor vs. ϵ

Figure 2 is a graph of $\left(\frac{u + u_t}{u}\right) \left(\frac{\epsilon - \epsilon_0}{\epsilon_0}\right) \epsilon$ against the per cent voids, and a straight-line plot is obtained for each different system.

Since the value of u_t can be predicted mathematically, the correlations shown in Figures 1 and 2 are useful in estimating the change of void fraction of a fluidized bed with changes in fluid velocity.

The slip velocity correction, meaningful only after the fluidization of solid particles has started, holds as ϵ approaches unity even in fully transported beds or settling particles.

The T factors for fluidization are

$$T_H = \frac{hD_p}{k} \Big/ \left(\frac{C_p \mu}{k}\right)^{1/3} \left(\frac{u + u_t}{u}\right) \left(\frac{\epsilon - \epsilon_0}{\epsilon_0}\right) \epsilon \tag{4}$$

$$T_D = \frac{k_g P_b M_m D_p}{D_v \rho} \Big/ \left(\frac{\mu}{\rho D_v}\right)^{1/3} \left(\frac{u + u_t}{u}\right) \left(\frac{\epsilon - \epsilon_0}{\epsilon_0}\right) \epsilon \tag{5}$$

These are also correlated against the superficial Reynolds number.

MOMENTUM TRANSFER IN GRANULAR BEDS

A pressure drop correlation, related to the heat and mass transfer factors, can be devised by assuming that the pressure drop in fixed beds or the weight gradient in fluidized beds is equal to the sum of the drag forces on the particles. For fixed beds

$$\frac{\Delta P}{L} = \frac{R \cdot n}{g_c} \tag{6}$$

where the drag on each particle is $R = \frac{1}{2}(C_D S \rho_f u^2)$. For fluidized beds

$$(1 - \epsilon)(\rho_s - \rho_f) = R \cdot n / g_c \tag{7}$$

which $= \frac{1}{2}(C_D S \rho_f u^2 n / g_c) = f_s' \times 6 (u^2 \rho_f / 2 g_c D_p)$.

Before the beginning of fluidization, the pressure drop has been shown by Leva (*12*) to vary as $(1 - \epsilon)/\epsilon^3$. This voidage function cannot be applicable as ϵ approaches unity. Lewis, Gilliland, and Bauer (*13*) have proposed that the void function is proportional to $\epsilon^{-4.65}$ for fluidized beds. Figure 3 indicates that the Leva function is essentially the same as the Lewis one and from this graph

$$F(\epsilon) = \frac{\epsilon^{-4.2}}{6} \tag{8}$$

Since for single particles, the void function is unity, the generalized void function was taken to be $6\,F(\epsilon)$.

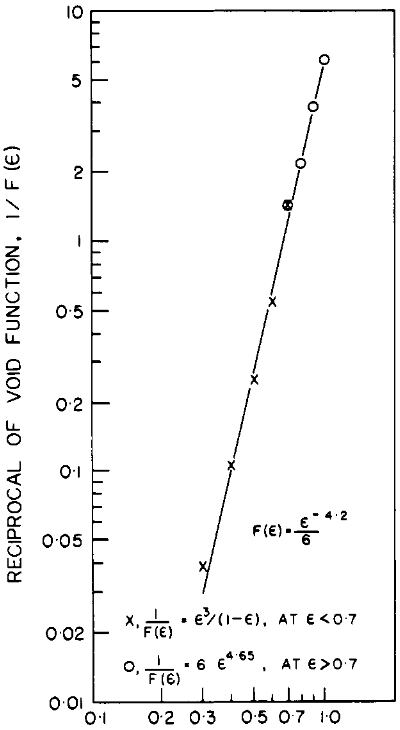


Figure 3. Void function vs. ϵ

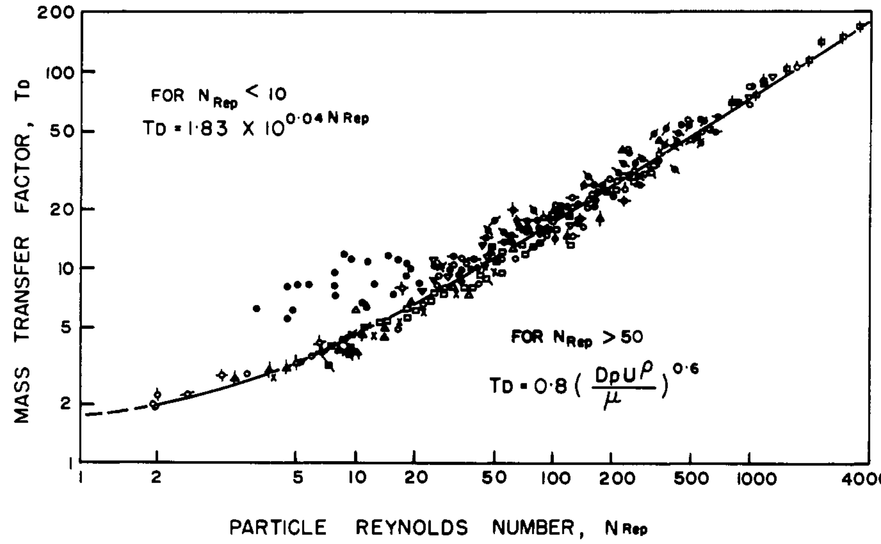


Figure 4. Mass transfer factor, T_D vs. N_{Rep}

Based on the original Chilton-Colburn analogy, T_M should be equal to $(f/2)N_{\text{Rep}}$. Accordingly, Equation 7 was rearranged to yield

$$\frac{\Delta P}{L_a} = \rho_s - \rho_f = T_M \times \frac{36}{g_c D_p^2 \phi_s^2} \times u \times \frac{\epsilon^{-4.2}}{6} \tag{9}$$

or

$$T_M = K \frac{6\epsilon^{4.2}}{u} \times \frac{\Delta P}{L_a} \tag{10}$$

for fixed beds and equals

$$K \frac{6\epsilon^{4.2}}{u} (\rho_s - \rho_f) \tag{11}$$

for fluidized beds where

$$K = \frac{g_c D_p^2 \phi_s^2}{36\mu}, \qquad \text{and} \qquad L_a = L(1-\epsilon)$$

SHAPE FACTORS

The shape factor, ϕ_s, has been incorporated into the statement of the momentum transfer factor. This is the shape factor proposed by Carman (*2*) to correlate the data for irregular particles with spheres:

$$A_p = \phi_s A_{sphere}$$

where $A_{sphere} = \pi D_p^2 (1-\epsilon)/(\pi/6) D_p^3 = 6(1-\epsilon)/D_p$ for unit volume of bed. To compensate for the portion of the particles area which is not available for heat or mass transfer, the shape factor ϕ_{ea} proposed by Gamson (*5*) is

$$q = hA_p\phi_{ea} V(\Delta t)_m$$

$$w = KA_p\phi_{ea} V(\Delta l)_m$$

where $A_p = 6(1-\epsilon)\phi_s D_p$.

This shape factor, when employed in both the rate equation and the modified Reynolds number, results in j factors for heat and mass transfer which fall on the same line as data for spheres. For data in the literature in which these shape factors have not been included, the proper corrections are necessary for the proposed correlation.

CORRELATION OF MASS TRANSFER DATA

Figure 4 is a plot of the mass transfer T_D factors calculated from the literature data of Hobson and Thodos (*8*, *9*), Ishino (*10*), Kolasta (*11*), McCune and Wilhelm (*14*), and Bar-Ilan and Resnick (*1*). The range of variables include particle diameters from 0.0493 to 0.53 inch, void volumes of 35 to 100%, Schmidt numbers of 1 to 2000, and particle Reynolds numbers to 4000. The particle shapes are mostly spheres, but cylinders and flasks are included also.

At particle Reynolds numbers above 50, the transfer rates vary as $(N_{\text{Rep}})^{0.65}$. At Reynolds numbers below 10, the slope of the correlation is small. Between 10 and 50, a transition region can be said to exist.

The general agreement of the data is good, with the exception of some of the data of Hobson and Thodos in the transition region.

HEAT TRANSFER DATA

Figure 5 is a plot of the heat transfer factors, T_H, calculated from the data of Mickley and Trilling (*15*), Taecker and Hougen (*19*), and Heertjes and McKibbins (*7*). Fixed beds and fluidized beds are covered. Because the heat transfer factors were calculated indirectly from humidification studies, a wide range of particle shapes including spheres, Raschig rings, and Berl saddles are included.

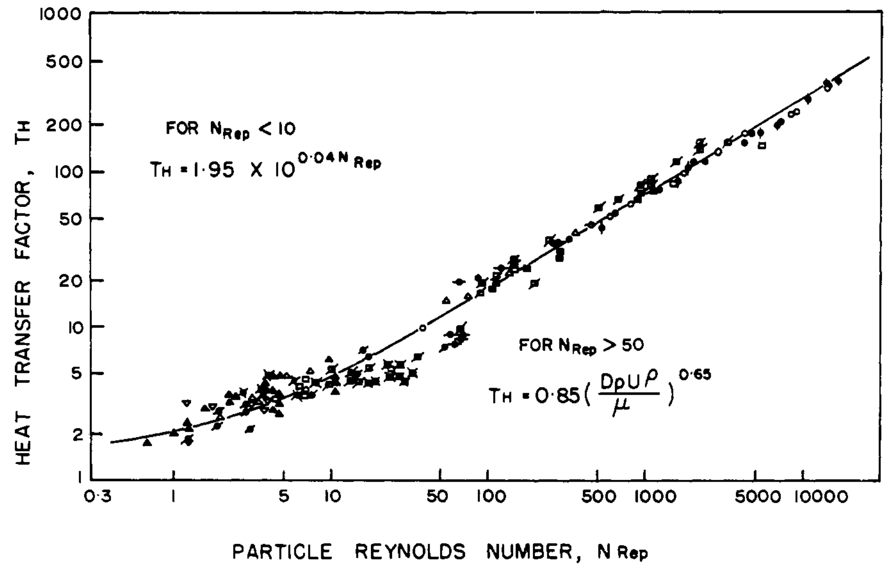
The published data for this plot are relatively meager, but the same correlation that was obtained for the mass transfer data holds.

PRESSURE DROP DATA

Figure 6 is a plot of the momentum transfer factor, T_M, calculated from the published data of Hariu and Molstad (*6*), McCune and Wilhelm (*14*), and Wilhelm and Kwauk (*21*). This plot includes data for a range of particle diameters for fixed and fluidized beds and for the settling of single particles. The data of McCune and Wilhelm include transported beds.

GENERALIZED CORRELATION

T_D, T_H, and T_M plotted as one curve (Figure 7) are representative of data from the previous three plots, but do not include all data. The data can be represented by the equations



← Figure 5. Heat transfer factor, T_H vs. N_{Rep}

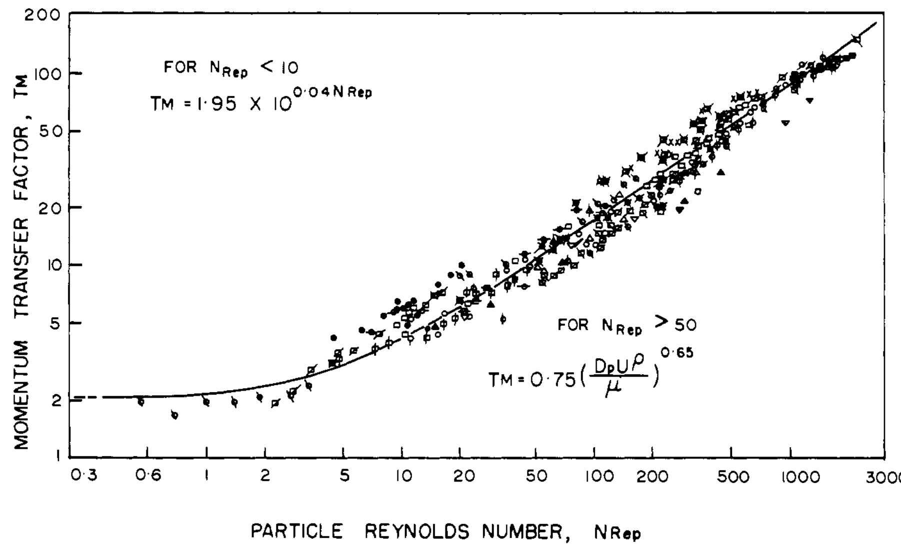
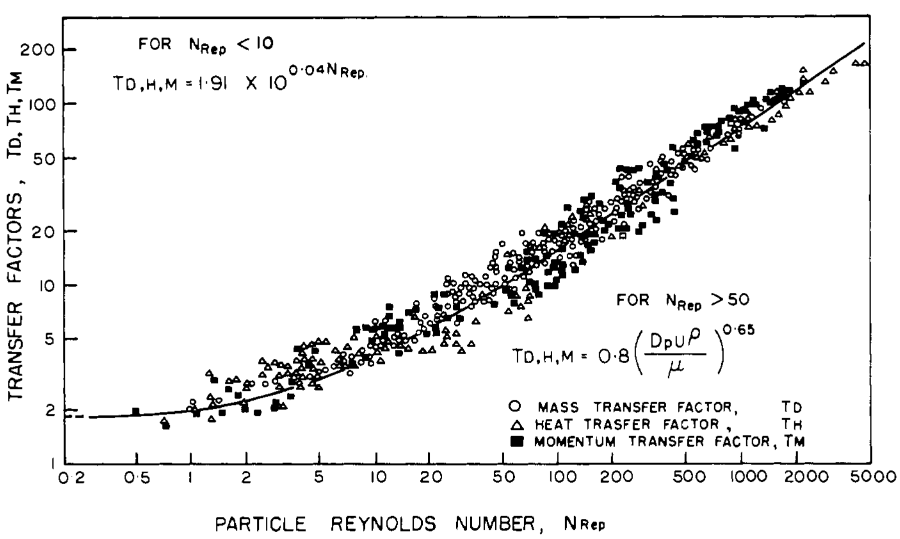


Figure 6. Momentum transfer factor, T_M vs. N_{Rep} →



← Figure 7. Generalized correlation of mass, heat, and momentum transfer in fixed, fluidized, and transported beds (single particles)

For $N_{Rep} > 50$ $\qquad T_D$, T_H, or $T_M = 0.8\ (D_p\rho u/\mu)^{0.65}$ (12)

For $N_{Rep} < 10$ $\qquad T_D$, T_H, or $T_M = 1.91 \times 10^{0.04 N_{Rep}}$ (13)

The maximum deviation in the data plotted is about 50% and the average deviation is about 10 to 15%.

There is essentially no separation between heat and mass transfer data, although Gamson (5) reported that j_H/j_D = 1.076. The correlation is probably not sensitive to the corresponding difference in the T factors.

HEAT AND MASS TRANSFER FACTORS AT LOW REYNOLDS NUMBERS

The T_D and T_H factors approach a value of 1.8 asymptotically as the particle Reynolds number decreases. For the evaporation of water from a drop in still air, the Schmidt and Prandtl numbers are 0.657 and 0.91, respectively. The values of the Nusselt numbers for heat transfer and for mass transfer to still air may be calculated from the following equations:

For heat transfer

$$N_{Nu} = T_H/(Pr)^{1/3} = 1.83/0.87 = 2.1$$

For mass transfer

$$N'_{Nu} = T_D/(Sc)^{1/3} = 1.95/0.91 = 2.14$$

These numbers agree with the theoretical values of Ranz and Marshall (*17*).

ACKNOWLEDGMENT

The author wishes to acknowledge his great indebtness to A.I. Johnson, University of Toronto, for his kind assistance throughout the course of the work and the preparation of this article.

NOMENCLATURE

A = area of heat or mass transfer, sq. ft.
A_p = surface area for one particle unit, sq. ft.
C = concentration of solute in fluid, lb. moles/cu. ft.
C_p = specific heat of fluid, B.t.u./lb. ° F.
C_D = $2R/\rho u^2 s$ = drag coefficient in Newton's equation (–)
D_p = particle diameter. Equivalent spherical diameter is used for shapes other than spheres, ft.
D_v = diffusivity coefficient of transferable component in gaseous state, sq. ft./hr.
f_s' = surface equivalent friction coefficient in granular bed (–)
$F(\epsilon)$ = function of (ϵ)
g_c = dimensional conversion constant (–)
G = actual mass velocity of flowing fluid, lb./hr. sq. ft.
h_g = heat transfer coefficient for gas film, B.t.u./hr. sq. ft. ° F.
j = transfer factor (–)
k = thermal conductivity, B.t.u./hr. ft. ° F.
k_g = mass transfer coefficient for gas film, lb. moles/hr. sq. ft. atm.
K = $\dfrac{g_c \cdot D_p^2 \cdot \phi_s^2}{36\mu}$ = constant
L = height of bed, ft.
L_a = $L(1-\epsilon)$, ft.
M_m = mean molecular weight of fluid, lb./mole
n = number of particles per unit volume of bed (–)
Nu = hD_p/k = Nusselt number (–)
Nu′ = $\dfrac{k_g P_b M_m D_p}{D_v \rho} = \dfrac{k_l C_{lf} M_m D_p}{D_l \rho}$ = mass transfer analogous to Nusselt number for heat transfer (–)
N'_{Re} = $\dfrac{D_p \rho \overline{V}_{sr}}{\mu}$ = hypothetical relative Reynolds number defined
P = pressure, lb./sq. ft.
ΔP = pressure drop, lb./sq. ft.
P = partial pressure of transferable component in gas film, atm.
P_b = mean partial pressure of the nontransferable component in gas film, atm.
Pr = $C_p \mu/k$ = Prandtl number (–)
R = resistance force, lb.
N_{Rep} = $\dfrac{D_p \rho_u}{\mu}$ = particle Reynolds number (–)
S = area projected by property against fluid flow, sq. ft.
Sc = $\mu/\rho D_F$ = Schmidt number (–)
T_H = heat transfer factor defined by Equation 1 or 4 (–)
T_D = mass transfer factor defined by Equation 2 or 5 (–)
T_M = momentum transfer factor defined by Equation 10 or 11 (–)
u = superficial fluid velocity based on empty column, ft./sec.
u_t = average terminal free falling velocity of particle in still fluid, ft./sec.
$\overline{V}_{s\infty}$ = $u_t + u$ = average slip velocity of particles in free vertical motion, ft./sec.
$\overline{V}_{sr}$ = average slip velocity of particles in a fluidized bed, ft./sec.
V = volume of one particle unit, cu. ft.

Greek

α = $\dfrac{u + u_t}{u}$ = dimensionless group
β = $\dfrac{\epsilon - \epsilon_0}{\epsilon_0}$ = dimensionless group
θ = $\alpha\beta\epsilon = \left(\dfrac{u+u_t}{u}\right)\left(\dfrac{\epsilon-\epsilon_0}{\epsilon_0}\right)\epsilon$ = state of fluidization factor (–)
ϵ_0 = maximum fractional void volume in fixed bed (–)
ϵ = fractional void volume in bed (–)
ρ_f = fluid density, lb./cu. ft.
ρ_s = solid density, lb./cu. ft.
ϕ_s = geometric shape factor by Carman (–)
ϕ_{eG} = shape factor by Gamson (–)
μ = fluid viscosity, lb./hr. ft.

Subscripts

d or D = mass transfer
f = fluid condition
g = gas phase
h or H = heat transfer
l = liquid phase
m = mean conditions for property under consideration
M = momentum transfer
p = particle
r = relative condition for property under consideration
S or s = solid, surface condition for property under consideration, or slip velosity

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RECEIVED for review July 14, 1960. Accepted May 12, 1961. Chemical Institute of Canada, 37th Annual Meeting, Kingston, Canada, March 1957. Part of this work was done at the University of Toronto, Toronto, Canada.